M-ary PSK Signal Power Spectrum at the Output of a Nonlinear Power Amplifier

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Abstract — **This paper proposes a simple accurate method to estimate the output wideband M-PSK spectrum, which is spreaded by a nonlinear amplifier. The amplifier is modelled by a complex power series extracted from CW measurements. Some papers [1,2] relate spectral regrowth assuming that the input signal is Gaussian. An other contribution [3] derives a closed form expression between the output covariance function and the input QPSK-OQPSK signal according to cumulant properties for a third order polynomial model. Even if the generalization to higher order is straighforward, high order cumulant statistic expressions are quite tedious and difficult to exploit. In this paper a simple closed-form expression between input signal and output signal spectrum that can be applied to any M-PSK signal for any order of a memoryless nonlinear model is derived. Different pulse waveforms are simulated to test this closed form expression.**

I. Introduction

The efficiency and the linearity of a power amplifier (PA) are the main goal in a wireless communications system. The more efficient is the power amplifier, the smaller is the battery. Besides, this entails a cheaper cooling system and extends speech time. A power amplifier is efficient when input signal is driven near the 1dB compression point. However, this will result in distorsion and intermodulation. Unless the signal is robust with respect to the nonlinearities (like modulation with constant envelope such as Minimum Shift Keying MSK used in the GSM system), its bandwidth is widened by the odd order non linearity (spectral regrowth). With amplitude envelope fluctuation, AM-AM and AM-PM distorsions are dispatched towards the output signal. However, standards impose strict constraints on ACPR (Adjacent Channel Power Ratio) and accurate methods for its determination are one of the main concerns in a RF power amplifier design. Gard [1] and Wu [2] give an expression of the spectrum based on gaussian input assumption In his paper, Wu also assumes that the AM-PM characteristics of the amplifier are constants. However, the assumption that input signal is gaussian is unrealistic for many digital communications signals. Zhou [3,6] proposes a closed form expression between the output covariance function and the high order statistics of the input QPSK-OQPSK signal according to cumulant properties for a third order polynomial model. However, even if the generalization to higher order is straighforward, high order cumulant statistic expressions are quite tedious. In this paper we present a simple accurate method to assess the average power density spectrum at the output of an amplifier modelled by a 2N+1-order complex polynomial model, driven by a M-ary PSK input signal. We are essentially interested in the analysis of the QPSK spectrum when the random sequence of the source is associated with different pulses.

II. Problem Formulation

III. The general transmit system can be simplified [4] as in figure 1. The form of the input signal is

$$e_1(t) = \sum_{k=-\infty}^{\infty} A_k \delta(t - kT_b)$$

where A_k is the digital transmitted sequence, T_b is the time between successive elements of the sequence. In figure 1, we define p(t) as the appropriate pulse waveform. In the literature, p(t) is often taken as a rectangular pulse of T_b seconds long, but a more realistic version of the idealized rectangular pulse train, like a trapezoï dal waveform may be considered. We will also analyse other waveforms to show the effects of the nonlinear amplifier on the output spectrum.

The signal at the output of the pulse waveform is

$$e_i(t) = \sum_{k=-\infty}^{\infty} A_k p(t - kT_b) \qquad (1)$$

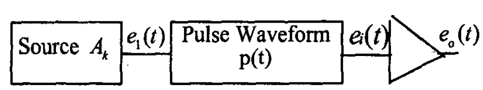

Figure 1 : block diagram of a communication chain.

The source produces a sequence of independent, identically distributed binary symbol. For a M-ary PSK source, $A_k = Ae^{j\phi_k}$ where $\phi_k = \pi \frac{k}{2^M - 1}$, $k \in [0, 2^M - 1]$.

This sequence is multiplied by the pulse waveform and is usually filtered by a raised cosine filter. The resultant

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signal is passed through an amplifier. The amplifier is generally modelled by a complex polynomial, whose coefficients are either estimated with the mean squares method according to CW measurements or assessed with the amplifier characteristics such as the gain, the compression point and the two-tone third order intercept point (IP3). Even if this last method does not give any information about the phase distorsion (AM-PM), it is commonly used. Indeed, first the characteristics of the amplifier are usually given in the data book and moreover no measu-rements are needed. But, as no phase is taken into account, the coefficients are real and they are limited by the number of characteristics.

Whatever the chosen methods, we assume here that the amplifier is memoryless. In this case, we can approximate its baseband input/output relationship by

$$e_o(t) = e_i(t)\sum_{n=0}^{N} a_{2n+1} \left|e_i(t)\right|^{2n} \qquad (2)$$

In the case of M-PSK signal, we can write

$$e_o(t) = e_i(t)\sum_{n=0}^{N} a_{2n+1} A^{2n} \left|\sum_{k=-\infty}^{\infty} e^{j\phi_k} \mathrm{p}(t-kT_b)\right|^{2n}.$$

Our aim is to analyse the power spectral density according to the baseband pulse waveform p(t)

IV. ANALYSIS.

The previous formulation can be expanded according to

$$\left|\sum_{k=-\infty}^{\infty} e^{j\phi_k} \mathrm{p}(t-kT_b)\right|^{2n} = \left|\sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_{2n}=-\infty}^{\infty} e^{j\theta} \mathrm{p}(t-k_1T_b)\cdots p(t-k_{2n}T_b)\right|$$

If we assume that p(t) is limited to $[-T_b/2, T_b/2]$ then, the expression can be simplified as

$$\left|\sum_{k=-\infty}^{\infty} e^{j\phi_k} \mathrm{p}(t-kT_b)\right|^{2n} = \sum_{k=-\infty}^{\infty} \mathrm{p}^{2n}(t-kT_b)$$

and, the output signal (2) can be expressed :

$$e_o(t) = \sum_{n=0}^{N} a_{2n+1} A^{2n+1} \sum_{k=0}^{\infty} \mathrm{p}^{2n+1}(t-kT_b) e^{j\phi_k} \qquad (3)$$

Filtered by the pulse waveform, the input signal of the amplifier can not be considered to be a stationary random signal. As a result, the power spectrum can be obtained as follows [5]. First, we compute the function

$$\Gamma_\xi(f_1,f_2) \stackrel{\Delta}{=} E\left[\Xi(f_1)E^*(f_2)\right]$$

with $\Xi(f)$ represents the Fourier transform of (3) and given by

$$\Xi(f) = \sum_{n=0}^{N} a_{2n+1} A^{2n+1} \sum_{k=-\infty}^{\infty} H_n(f) e^{-j2\pi k f T_b} e^{j\phi_k}$$

Substituting our model to $\Gamma_\xi(f_1,f_2)$, we obtain :

$$\Gamma_\xi(f_1,f_2) = \sum_{m,n=0}^{N} \sum_{k,k'=-\infty}^{\infty} a_{2m+1} a_{2n+1} A^{2(m+n+1)} H_m(f_1) H_n^*(f_2) \, e^{-j2\pi(f_1 k - f k')T_b} E(e^{j\phi_k} . e^{-j\phi_{k'}})$$

with $H_n(f) = H(f) \otimes \cdots \otimes H(f)$ (2n+1 times convolution between $H(f)$).

Then, we can write $\Gamma_\xi(f_1,f_2)$ in the form

$$\Gamma_\xi(f_1,f_2) = G_\xi(f_1)\delta(f_1-f_2) + \Delta_\xi(f_1,f_2)$$

where $G_\xi(f)$ is the required spectrum

Noting :

$$\Gamma_\xi(f_1,f_2) = F(m,n,f_1,f_2) \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \alpha_l e^{-j2\pi f_1 l T_b} e^{-j2\pi(f_1-f_2)kT_b}$$

where

$$F(m,n,f_1,f_2) = \sum_{m=0}^{N} \sum_{n=0}^{N} a_{2m+1} a_{2n+1} A^{2(m+n+1)} H_m(f_1) H_n^*(f_2)$$

$\alpha_l = E(e^{j\phi_k} e^{-j\phi_{k'}})$ with $l = k - k'$.

Noting that the Fourier transform of a comb is a comb, we have the following relation

$$\sum_{n=-\infty}^{\infty} e^{-j2\pi(f_1-f_2)nT_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta(f_1 - f_2 - \frac{n}{T_b})$$

then, we can express $\Gamma_\xi(f_1,f_2)$ as

$$\Gamma_\xi(f_1,f_2) = \frac{F(m,n,f_1,f_2)}{T_b} \sum_{l=-\infty}^{\infty} \alpha_l e^{-j2\pi f_1 l T_b} \sum_{k=-\infty}^{\infty} \delta(f_1 - f_2 - \frac{k}{T_b})$$

According to this function, we can extract the required spectrum for n=0 and $f = f_1 = f_2$. As a result, we find :

$$G_\xi(f) = \frac{F(m,n,f,f)}{T_b} \sum_{l=-\infty}^{\infty} \alpha_l e^{-j2\pi f l T_b}$$

Assuming that the sequence produced by the source is independent and uniformely distributed on the unity circle, we obtain :

$$\alpha_l = \begin{cases} 1 & if\ l = 0 \\ 0 & if\ l \neq 0 \end{cases}$$

Finally, the power spectrum is :

$$PSD = \frac{1}{T_b} \left|\sum_{n=0}^{N} a_{2n+1} A^{2n+1} H_n(f)\right|^2 \qquad (4)$$

For N=1 and 2, we have found the expression given in [6,7]. But the proposed method is completely different from the methods proposed in [6,7] and give a more general result.

V. SIMULATION

In order to test our derivation, several simulations are performed. The source delivers a sequence of 1024 QPSK symbols which are filtered by a rectangular waveform pulse, or by a cosine waveform pulse. The sampling period is $T_b/8$. A linear gain is added before the amplifier to modify the average input power. The model of the nonlinear amplifier is extracted from measurement on amplifier built by Alcatel and summed up on table 1. As the phase is taken into account, the first coefficient is negative.

When the waveform pulse is rectangular (figure 1), no effects can be observed, since the envelope of the input signal is constant. The same spectrum is observed at the input and the output (solid line) of the amplifier and good agreement is achieved with theoretical (4) spectrum (dashed line), as we can see in figure 1. We note that for any k and with a rectangular waveform, $h^k(t)=h(t)$.

So, we obtain

$$H_k(f)=T_b\mathrm{sinc}(\pi fT_b)$$

Consequently, the output spectrum is

$$PSD_{output}=\left|\sum_{k=0}^{N}a_{2k+1}A^{2k+1}\right|^2 T_b\mathrm{sinc}^2(\pi fT_b)$$

and the input spectrum is :

$$PSD_{input}=A^2T_b\mathrm{sinc}^2(\pi fT_b)$$

in the following simulation, the pulse waveform is taken as cosine function.

$$h(t)=\begin{cases}\sqrt{2}\cos(\pi t/T_b) & |t|<T_b/2\\ 0 & |t|>T_b/2\end{cases}$$

Figure 2 presents the theoretical output spectrum and the output spectrum estimated when the amplifier is modelled by a_1 and a_3, and for a nonconstant envelope (a cosine pulse waveform). Close agreement between the two case is observed.

Figure 3 and 4 show the importance of an accurate model. In both figures, the theoretical spectrum is calculated by the equation 3 with N=2 (order 3). In fig 3, the amplifier is also modelled by a third order memoryless polynomial, but in figure 4, the model of the amplifier is more accurate (order 11^{th}). The dotted line is the input spectrum, the dashdot line is the theoretical spectrum limited at the 3^{th} order, whereas the solid line is the output spectrum of the amplifier.

Coefficients	value
a_1	-33.8790 -37.6302i
a_3	-0.0020 - 0.0111i
a_5	8.9246e-006 +6.7661e-006i
a_7	-3.8122e-009 -1.8921e-009
a_9	6.7910e-013 +2.8319e-013i
a_{11}	-4.2630e-017 -1.6363e-017i

Table 1

VI. CONCLUSION

An accurate formulation has been proposed to evaluate the power spectrum of the output of a power amplifier driven by a M-PSK signal. Under the assumption of a memoryless amplifier and a time-limited pulse waveform, our work shows the effects of the nonlinearity on M-PSK signal. We are currently working on analysing the effects of spectral regrowth taking into account the effects of the filter or of the memory.

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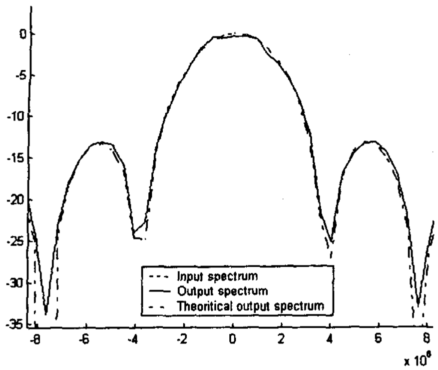

Figure 1 : Rectangular Waveform

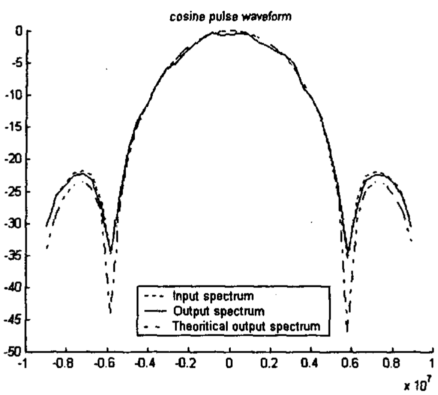

Figure 2 : Cosine pulse waveform low power

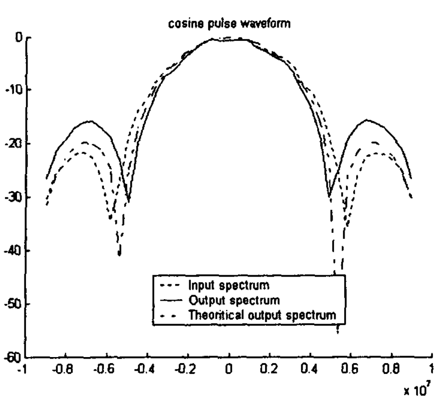

Figure 3 : Cosine pulse waveform where input power is near the compression point

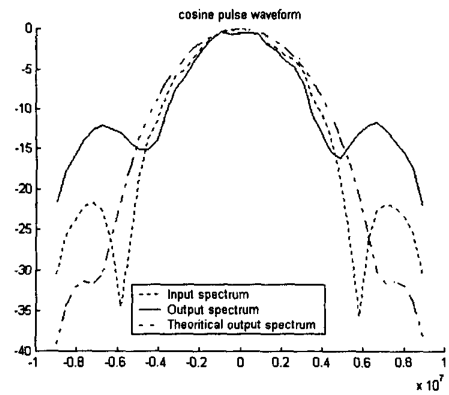

Figure 4 : Cosine pulse waveform where input power is near the compression point and the order of the theoretical model polynomial is limited compared to the true order of the nonlinear model